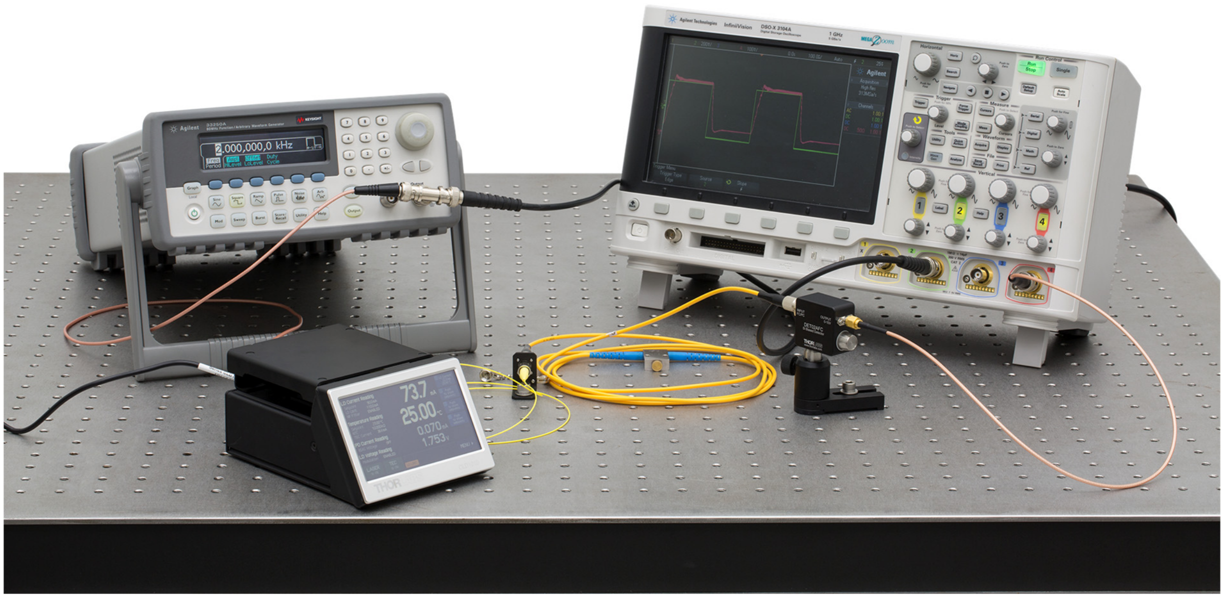
THORLABS

Lab Fact



System Bandwidth and Pulse Shape Distortion

This Lab Fact investigated the distortion of signals output by a system with limited 3 dB bandwidth. The input signals were inherently broadband, periodic rectangular pulse trains with different duty cycles and repetition rates. A function generator supplied the input signals, and the system included a CLD1010LP Combined Mount and Current and Temperature Controller for Fiber-Pigtailed Laser Diodes, a fiber-pigtailed laser diode, a DET02AFC FC / PC Coupled Photodetector, and a 1 GHz oscilloscope.

The 3 dB bandwidth of the system was found from its frequency response magnitude. After measuring the frequency-dependent scaling factors relating the amplitudes of the input and output signal components, the 3 dB bandwidth was identified as the range of frequencies over which the normalized scaling factors were at least 0.707.

Results of increasing the input signal's repetition rate was shown using Fourier series analysis and demonstrated experimentally. The number of input signal components with frequencies above the system's 3 dB cutoff frequency increased with repetition rate, and the subsequent strong attenuation of these components in the output signal resulted in output pulse distortion. It was also shown that decreasing the duty cycle of the input signal increased output pulse distortion. This work demonstrated the importance of considering the impact of the system's 3 dB bandwidth, the input signal's repetition rate, and the input signal's duty cycle on the distortion of the output signal.

December 31, 2018

1 *Introduction and Background*

Rectangular pulse trains are used in a wide variety of test and measurement applications in both research and industry. When the input to a system is a rectangular pulse train, often the desired output signal is also a rectangular pulse train. One requirement for obtaining a high-fidelity output signal is compatibility between the bandwidths of the system and the input signal. Insufficient system bandwidth will result in a distorted output signal, with less-abrupt transitions and broader features than in the input signal. In an extreme case of insufficient bandwidth, an input rectangular pulse train would be output as a sine wave.

During this Lab Fact investigation, a series of rectangular pulse trains from a function generator were input to a laser diode controller, whose modulated output current signals were used to drive a fiber-pigtailed laser diode. The modulated optical intensity signals from the laser diode were detected and compared with the rectangular pulse trains from the function generator using Fourier series and time-domain approaches. This analysis was used to explore the influence of the repetition rate of the input signal, the duty cycle of the input signal, and the bandwidth of the system on the distortion of the measured optical output pulses.

1.1 Fourier Series Expansion of Rectangular Pulse Trains

An ideal periodic rectangular pulse train is shown in Figure 1. The period (T) determines the pulse spacing. The signal instantaneously transitions between low and high states, whose amplitudes are 0 and X, respectively.

The fractional duty cycle (κ) is the fraction of time the signal dwells in the high state. The width (κT) of each ideal rectangular pulse is the product of the period and the fractional duty cycle. The user specifies the duty cycle, repetition rate, amplitude, and DC offset of the rectangular pulse train based on the requirements of the application.

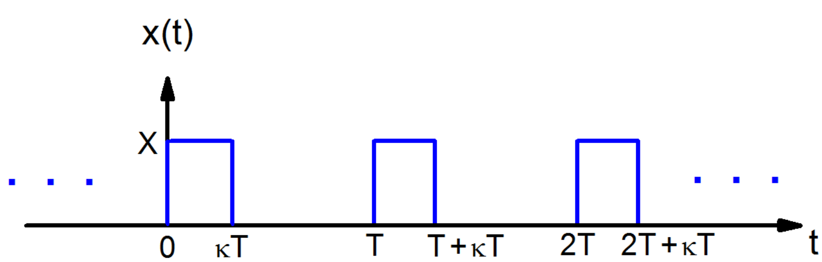


Figure 1 *Ideal train of rectangular pulses, with instantaneous transitions between low and high signal states at perfectly regular intervals.*

Waveforms, such as rectangular pulse trains, that are periodic in time (t) can be represented using a Fourier series[1], which includes sums of mutually orthogonal sine and cosine functions,

$$x(t) = \frac{A_o}{2} + \sum_{n=1}^{\infty} A_n \cos(2\pi n f_o t) + \sum_{n=1}^{\infty} B_n \sin(2\pi n f_o t) + C \quad . \tag{1}$$

The first term in Eq. 1 is a constant calculated using the DC component coefficient (A_o). The summations in the second and third terms extend to infinity. Each sine and cosine function in these summations is multiplied by a Fourier coefficient (A_n or B_n, respectively), which depends on the index of summation (n). The coefficients for a rectangular pulse train,

$$A_o = \kappa X \quad , \tag{2a}$$

$$A_n = \frac{X}{n\pi}[\sin(2\pi n\kappa)] \quad , \tag{2b}$$

and

$$B_n = \frac{X}{n\pi}[1 - \cos(2\pi n\kappa)] \quad , \tag{2c}$$

are calculated using signal parameters. All coefficients are proportional to the high state amplitude. The values of A_n and B_n are strongly dependent on the index and the fractional duty cycle. Their product is in the arguments of the sine and cosine functions in Eq. 2b and 2c, and both coefficients also have n in their denominators.

The constant (C) in Eq. 1 controls the DC offset of the rectangular pulse train. In this work, C corresponds to the constant component of current used to drive the laser diode, as described in Section 2. For the discussion in this section, C was set to zero and X was set to one.

In the summations included in Eq. 1, the oscillation frequency of each trigonometric term is the fundamental frequency (f_o) multiplied by n, which is a positive integer. The terms of each summation compose a harmonic series. The first harmonic frequency is equal to the fundamental frequency: the first harmonic frequency is found by multiplying the fundamental frequency by one, the index of the term.

The repetition rate of a rectangular pulse train and the fundamental frequency of its Fourier series expansion are equal. Therefore, the repetition rate also equals the first harmonic frequency.

Modeled Rectangular Pulses

Each of the six plots in Figure 2 compares an ideal rectangular pulse (dashed-black outline) of a 20% duty cycle pulse train with a different modeled version (blue curve) of it. Eq. 1 was used to calculate the modeled waveforms, and the maximum index used in the calculations ranged from one to nine.

The modeled waveforms were used to define the high and low states of the ideal pulses. The high and low state amplitudes equaled the minimum and maximum amplitudes of the modeled

[1] Ferrel G. Stremler, *Introduction to Communication Systems, Third Edition*, Addison-Wesley Publishing Company, New York, 1990.

pulses, unless the modeled pulse oscillated around a minimum or maximum value. In that case, the average over that span defined the low or high state amplitude, respectively. For example, in Figure 2 (b) the low state was defined to be the average of the oscillations on either side of the peak. As the maximum amplitude of the modeled peak did not oscillate, the high state value was defined to be the maximum amplitude of the peak.

The relative timing between the ideal pulse and modeled waveform was arranged so that the intersection between the low state and rising edge on each pulse coincided. The lowest-amplitude red circle marks this point. This alignment of the two waveforms was intended to facilitate the task of comparing the width of the modeled pulse's rising edge and the width of the ideal pulse. As discussed in Section 1.3, comparing these two parameters is the basis of one method used to characterize pulse shape. The distance separating the two red circles on the modeled waveform is equal to either the width of the modeled pulse's rising edge or the width of the ideal pulse, whichever is shorter. If the second red circle intersects the high state of the ideal pulse, the width of the rising edge of the modeled pulse is equal to or shorter than the ideal pulse width.

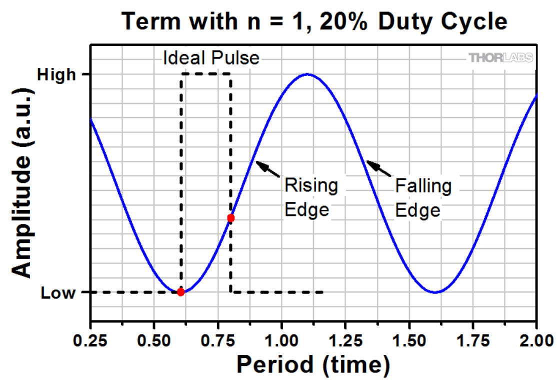
Including higher-frequency terms narrows the width of the modeled pulse and increases the steepness of its rising and falling edges. The slope on the rising edge of the modeled pulse shown in Figure 2 (a) is so shallow that the pulse does not reach the high state amplitude within the ideal pulse duration. Including terms with indices one and two in the model, as was done when calculating the modeled pulse shown in Figure 2 (b), results in a narrower pulse. Although the transition time between the low and high state amplitudes is still longer than ideal pulse width, the difference is small.

When terms with indices through the third are included in the model, as is the case in Figure 2 (c), the width of the rising edge of the modeled pulse is less than the width of the ideal pulse. The amplitude of the modeled pulse's low-state oscillations have also decreased.

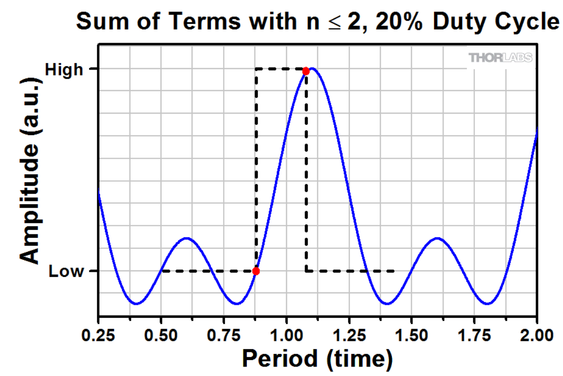
As shown in Figure 2 (d), one of the most notable effects of including terms with indices through the sixth in the modeled pulse is a narrowing of the pulse width. This, combined with the reduced amplitude of the low-state oscillations, results in improved peak definition when compared with models that include fewer harmonic frequency terms.

Adding terms with the next index, for a total of seven, to the model has a significant impact on the appearance of the modeled pulse, shown in Figure 2 (e). The peak of the pulse is flatter, and the width of the rising edge of the pulse is shorter.

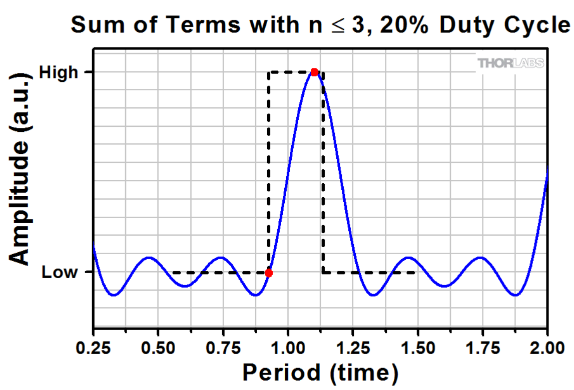
When terms with indices through the ninth are included in the modeled pulse, its width is a better match to the width of the ideal pulse. This is shown in Figure 2 (f). The amplitude of the low-state oscillation is smaller, and the width of the rising edge of the pulse is less than half the duration of the ideal pulse.



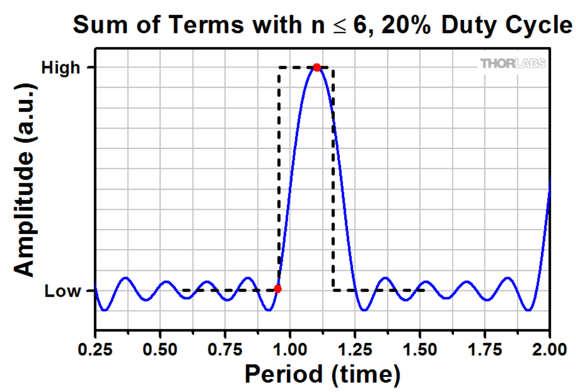
(*a*) *The model included the term with index one. The modeled waveform is sinusoidal.*



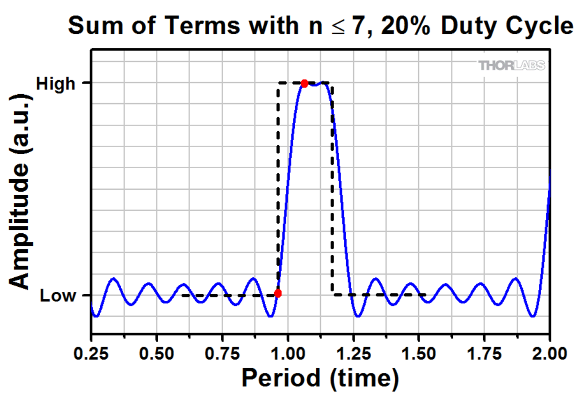
(*b*) *The model included terms with indices one and two, and the pulse width is narrowed.*



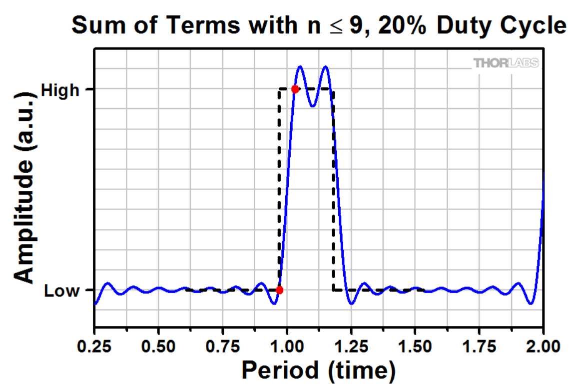
(*c*) *The model included terms with indices one through three, and the pulse is further defined.*



(*d*) *The model included terms with indices one through six. The term with index five is zero.*



(*e*) *The model included terms with indices one through seven. Maximum amplitude is reached in less than half an ideal pulse duration.*



(*f*) *The model included terms with indices one through nine and its width is closest to the width of the ideal pulse.*

Figure 2 *The six blue curves plotted above are different models of an ideal, 20% duty cycle rectangular pulse train. A time duration equal to 1.5 periods of the waveforms is shown for each. The dashed black outlines show the ideal pulse profile. The modeled pulse becomes better defined and more narrow with steeper sides as more terms of the Fourier series representation in Eq. 1 are included in the model.*

Figure 2 illustrates that the duty cycles of the ideal and modeled waveforms may be different. An ideal rectangular pulse train may have a duty cycle between 0 and 100%. When its duty cycle is not 50%, the duty cycle of the modeled waveform will become closer to 50% as fewer higher-index terms are included in the calculation. A signal that retains only the first term in the expansion, as is the case in Figure 2 (a), has a 50% duty cycle, since a sinusoid's pulse width always equals half of its period.

1.1.1 Square Waves

Constraining the duty cycle to 50% simplifies the Fourier series representation of Eq. 1 considerably. The result,

$$x(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n\,=\,1,\,3,\,5\ldots}^{\infty} \frac{1}{n} sin(2\pi n f_o t) \quad , \tag{3}$$

states that a square wave can be represented exclusively by a DC term and an infinite series of sine functions. In addition, only the harmonic terms with odd indices have non-zero amplitudes. The first five non-zero sinusoidal terms are plotted individually in Figure 3.

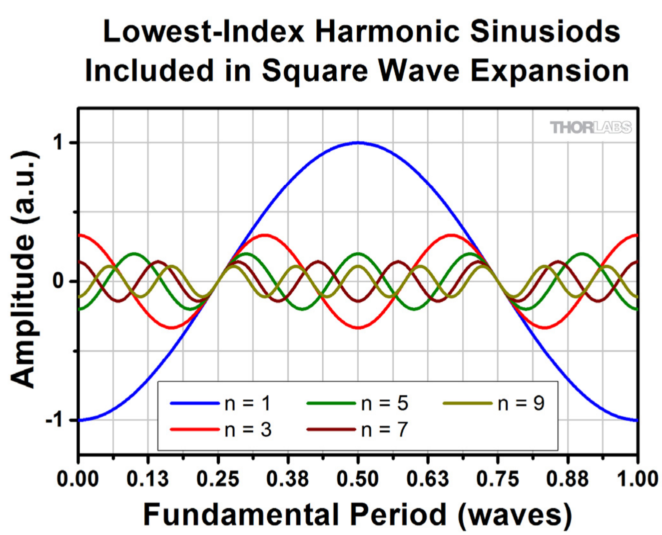
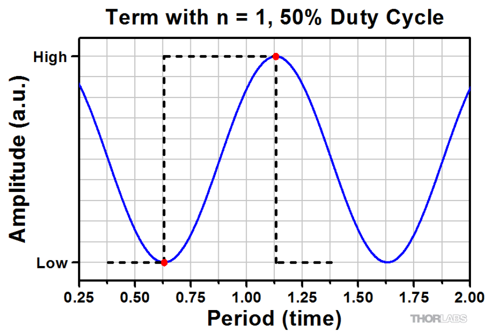
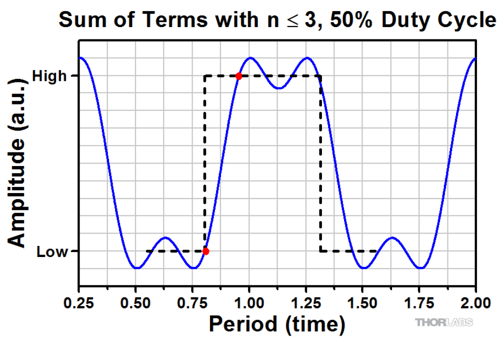


Figure 3 *The first five non-zero harmonic series terms in the Fourier series expansion of a square wave include the fundamental term* (n *= 1*)*, whose frequency matches the signal's repetition rate. The frequencies of the higher-index terms are integer multiples* (>*1*) *of the fundamental.*

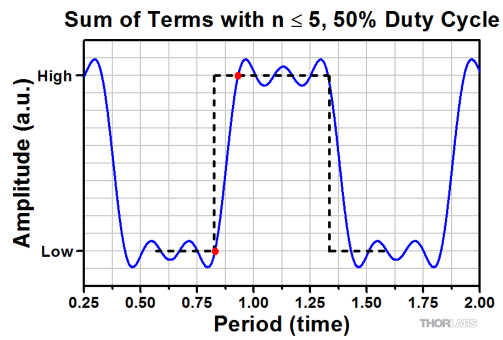
Approximations of the square wave were calculated by summing limited numbers of the terms given by Eq. 3. Waveforms resulting when terms with indices up to nine were included in the calculations are plotted in Figure 4. The blue curves are the modeled waveforms, and the dashed-black outlines are the ideal pulse profiles. As in Figure 2, the high and low state amplitudes equaled the minimum and maximum amplitudes of the modeled pulses, unless the modeled pulse oscillated around a minimum or maximum value. In that case, the average over that span defined the low or high state amplitude, respectively. The red circle with the lowest amplitude marks the boundary point between the low state and rising edge on both pulses, and the red circle with the highest amplitude marks the point at which the rising edge of the modeled pulse intersects the high state of the ideal pulse.



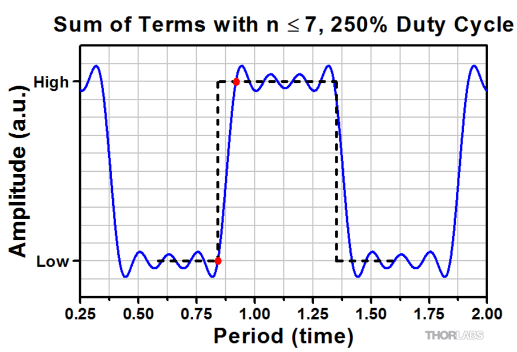
(*a*) *The model included the term with index one, whose transition time equals the ideal pulse width.*



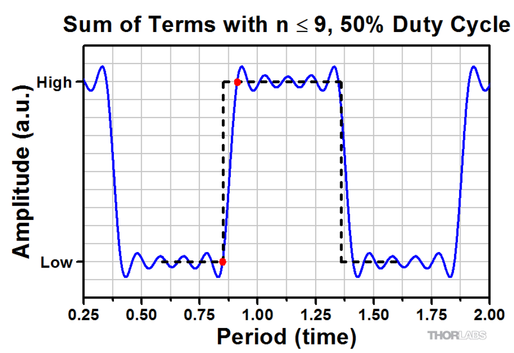
(*b*) *The model included terms with indicies one and three, and the pulse profile is recognizable.*



(*c*) *The model included terms with indices through the fifth, and the rising edge had a steeper slope.*



(*d*) *The model included terms with indices through the seventh, resulting in a more angular profile.*



(*e*) *The model included terms with indices through the ninth and had a recognizably rectangular profile.*

Figure 4 *Different models of an ideal square wave are plotted over 1.5 periods, and the dashed black outlines show the ideal pulse profile. As the signal and the term with index one have a 50% duty cycle, the modeled pulse is a better match to the ideal for every case, as compared with the modeled 20% duty cycle waveforms.*

Since both sine waves and square waves have 50% duty cycles, the first term of the Fourier series expansion, plotted in Figure 4 (a), has the same duty cycle as the ideal square wave. A consequence is that the transition time of this modeled pulse equals the ideal pulse width. This is in contrast to the modeled pulse shown in Figure 2 (a), in which a 20% duty pulse was modeled while limiting the index of the expansion to one. In that case, the width of the rising edge of the sine wave exceeded the width of the ideal pulse.

The modeled 20% and 50% duty cycle pulses can be compared when the models include expansion terms up to the same index. Under this constraint, the ratios of modeled to ideal pulse widths are always closer to unity for the modeled 50% duty cycle pulses. However, in both cases, models including only expansion terms with a maximum index of three resulted in recognizable pulses. In addition, when terms with indices up to and including the ninth are included, the modeled pulses in both cases have widths reasonably similar to the ideal case and profiles that are recognizably rectangular.

1.2 Device Frequency Response and 3 dB Bandwidth

One conclusion that can be drawn from Section 1.1 is that removing frequency components from a waveform results in distortion. However, some amount of distortion in an output signal may be unavoidable. Information about the frequency response of the system or device is helpful in evaluating whether its output signal will be suitable for an application.

Data characterizing the frequency response of a system over a range of frequencies may be supplied. These data may characterize both phase and magnitude responses, or only the magnitude response. When the system is designed to provide an output signal that is a scaled version of an input signal, the uniformity of the response and the maximum supported input signal bandwidth are of interest.

The phase response specifies the relative phase shift the system adds to each frequency component of the input signal. It is typically desirable for the phase response to be flat over the frequency range of interest. If the phase response varies over the input signal's bandwidth, the phase relationship between different frequency components in the input signal will not be preserved in the output signal, and the output signal will be distorted. The phase response was not considered in this work.

The magnitude of the frequency response specifies the relative scaling factors the system applies to the amplitudes of the input signal's frequency components. If the magnitude of the frequency response is not provided, it can be found by measuring the peak-to-peak output signal amplitude while incrementing the frequency of an input sine wave. The function generator, or other input signal source, should provide sinusoids with the same peak-to-peak amplitude over the entire frequency range of interest. At each input signal frequency, this frequency and the measured peak-to-peak amplitude of the output signal should be recorded as a pair. The measured output signal amplitudes are typically normalized with respect to the value measured at a reference frequency. Often the reference amplitude value is the maximum value, and the resulting scaling factors are between zero and one. A plot of the frequency response for a low pass filter is shown in Figure 5.

The 3 dB bandwidth specifies the range of input signal frequencies corresponding to normalized amplitude scaling factors greater than 0.707, assuming the value at the reference frequency is one. Multiplying the amplitude of an electrical signal by 0.707 reduces its power by half.[2] When the frequency response of a device or system is highest at low frequencies, as is the case in this work, the 3 dB bandwidth corresponds to the lowest frequency at which the magnitude response drops to a value 70.7% of its reference value. Beyond the 3 dB bandwidth, scaling factors are lower and the attenuation is greater.

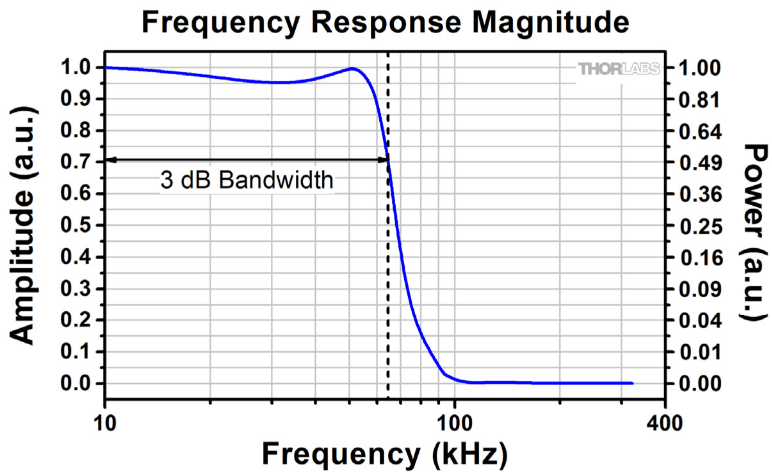


Figure 5 *The frequency response magnitude of a low-pass filter has high scaling factors at lower frequencies and low scaling factors at higher frequencies. The 3 dB bandwidth is indicated.*

The 3 dB bandwidth parameter is useful for quickly determining whether the device or system has the necessary bandwidth to provide a high-fidelity output signal for a given input signal. If frequency components in the input signal are greater than the 3 dB bandwidth, they may be significantly attenuated, which could result in unacceptable distortion of the output signal.

1.3 Quality Factor for Characterizing Rectangular Pulses

Modeling periodic signals using a Fourier series representation is a powerful and informative approach to analyzing signal distortion, but faster and less computationally intensive approaches are also of interest. The method presented in this section computes a quality factor that rates the similarity of a pulse's shape to two ideal cases: a perfect rectangular pulse and a pure $\sin(x)$ function from $x = -\pi/2$ to $3\pi/2$. This approach was developed to describe the shape of output signal pulses when:

- The input signal is a periodic waveform with nominally rectangular pulses.
- The duty cycle of the input signal is ≤50%.
- Lower frequency components have been minimally attenuated in the output signal.
- Higher frequency components may have been attenuated in the output signal.

[2] When the signal is a voltage or current, the signal power is directly proportional to the square of the signal amplitude. A drop to a normalized amplitude of 0.707 corresponds to a drop in power by a factor of $(0.707)^2 = 0.5$. Signal power is often specified on a logarithmic scale in terms of decibels (dB). Using this scale, the 0.707 normalized drop in amplitude corresponds to $10\log[(0.707)^2] = -3$ dB. In the phrase "3 dB bandwidth" the negative sign is typically omitted, as it is understood.

1.3.1 Characteristics of Ideal Rectangular Pulse Trains and Sinusoids

An ideal rectangular pulse train can be modeled, as discussed in Section 1.1, by computing and summing the infinite number of Fourier series expansion terms. Figure 6 shows a full period of a rectangular pulse train with a 50% duty cycle, which is also known as a square wave. The second curve, a sine wave, included in this figure results when the maximum index of the expansion is limited to one. The rectangular pulse train and the sine wave represent extreme cases at either end of a continuum of possible output waveforms. The time to transition between minimum and maximum amplitudes is a notable difference between the two.

Pulses in rectangular pulse trains are characterized by infinitely short transition times between low and high state amplitudes. The dwell time at the high state amplitude equals the pulse width, which may be any fraction of the period.

For a sinusoid to transition between minimum and maximum amplitudes, its argument must change by 180°, or π, which is equal to half of the period. The full-width-half-maximum pulse width also equals half of a period, and the dwell times of the pulse at the minimum and maximum amplitudes are infinitely short.

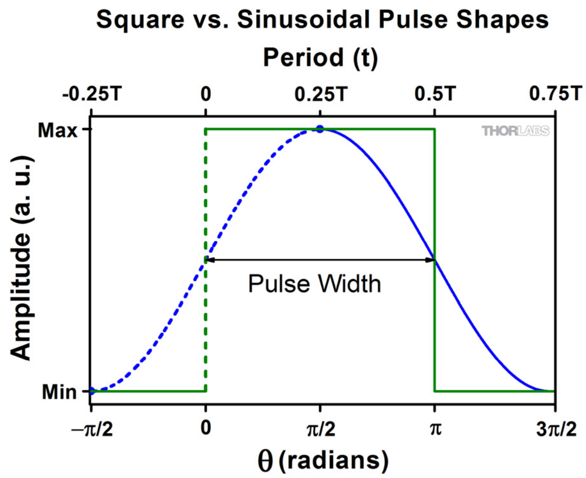


Figure 6 *The dashed edges of the square waveform (green) and sine wave (blue) show their rising edges. Both have the same pulse width. The transition time for the square wave is infinitely short, but the sine wave requires half a period to complete the transition.*

The time required for a pulse to transition between low and high amplitudes can be used as a basis for describing pulse distortion, as is described in the next section.

1.3.2 Derivation of Quality Factor

The quality factor approach was developed as a tool to assess, describe, and predict the shape of nominally rectangular output pulses, given the duty cycle and the repetition rate of the input pulse train. This approach assigns a numerical value to a pulse shape based on the steepness of its rising edge and the width of the ideal pulse it approximates.

The maximum value of the quality factor is one, and it is assigned to pulses identical to the ideal rectangular pulse, with the same pulse width and infinitely short transition times between low and high signal states. A quality factor of zero is assigned to pulses that require exactly the width of the ideal rectangular pulse to transition between low and high amplitude states. The modeled pulse plotted in Figure 4 (a) has a quality factor of zero. Negative quality factors are assigned to pulses whose rising edges are wider than the ideal pulse width. An example of a modeled pulse with a negative quality factor is plotted in Figure 2 (a).

The expression for the quality factor was derived from the transition times and pulse widths (W) of ideal rectangular and sinusoidal pulse shapes. The full transition times of real pulses can be challenging to measure, since the boundaries separating the low signal state, the rising edge of the pulse, and the high signal state can be difficult to identify. In recognition of this, the derivation follows the common practice of using rise time, defined here as the time separating positions 10% and 90% up the rising edge of the pulse, instead of the full transition time.

The sinusoidal pulse shape used in the derivation is shown in Figure 7. The amplitude is expressed as a function of radians, rather than of time or distance. Minima occur at -π/2 and 3π/2, and the maximum occurs at π/2. The radial pulse width (W_θ) is π.

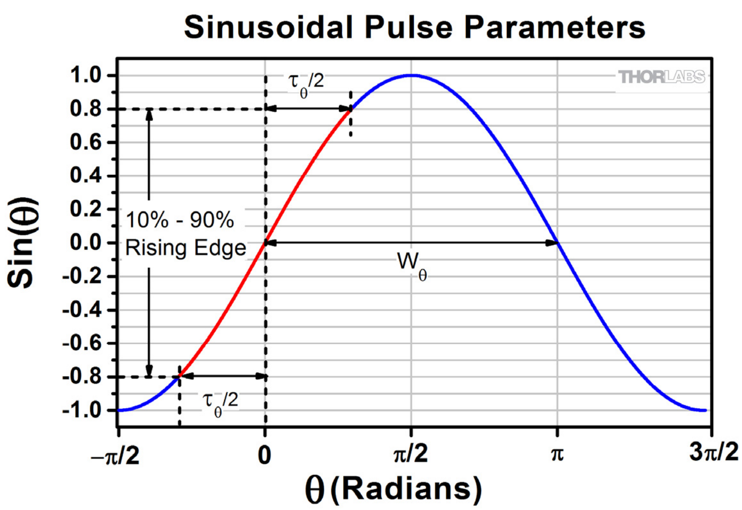


Figure 7 *Radial rise time and pulse width are shown defined with respect to a sine function.*

As the amplitude varies over ±1, the low state is -1 and the high state is +1. Radial rise time (τ_θ) corresponds to the radial distance separating the points at 10% and 90% of the full amplitude. These points correspond to amplitudes -0.8 and 0.8, whose radial separation,

$$\tau_\theta = 2sin^{-1}(0.8) = 1.85\,rad \quad , \tag{4}$$

equals the radial rise time. The ratio of radial pulse width to radial rise time,

$$\frac{W_\theta}{\tau_\theta} = \frac{\pi}{1.85\,rad} \approx 1.7 \quad , \tag{5}$$

is then used with the rectangular pulse parameters to define the quality factor (Γ),

$$\Gamma = 1 - \frac{W_\theta}{\tau_\theta}\frac{\tau_r}{W} \approx 1 - 1.7\frac{\tau_r}{W} \quad , \tag{6}$$

in which:

$\Gamma = 1$ for an ideal rectangular pulse shape.

$\Gamma = 0$ when the pulse shape is sinusoidal.

1.3.3 Dependence of Quality Factor on Duty Cycle

The maximum quality factor is always one, but the lowest possible quality factor depends on the difference between the ideal duty cycle of the input waveform and the 50% duty cycle of the sinusoid resulting when the index of the expansion is limited to one.

Quality factors of zero ($\Gamma = 0$) can occur only if the input waveform's duty cycle is ≤50%. When the input duty cycle equals 50%, zero is the lowest possible quality factor. Zero quality factors result when the time required by the output pulse to transition between low and high state amplitudes equals exactly the full width of the ideal rectangular pulse. Figure 4 (a) shows a pulse with a zero quality factor. The models of the 20% duty cycle case in Figure 2 indicate that a pulse with a zero quality factor would have a rising edge between that in Figure 2 (b) and (c).

Negative quality factors, which can occur only if the input duty cycle is <50%, are assigned to output pulses that cannot achieve the high state amplitude within the ideal pulse duration. Under these conditions, only a few of the lowest-index terms are included in the calculations of the modeled pulse or retained in the output signal. According to Eq. 1 and Eq. 2, the amplitudes of these lowest-index terms can be substantially smaller in magnitude than the amplitude of the ideal pulse. Measuring the rise times of these reduced-amplitude, distorted output pulses can be problematic. Their low amplitudes may also adversely affect some applications, such as those that rely on detection of specific pulse amplitudes to trigger an event. The pulse in Figure 2 (a) has a negative quality factor.

Quality factor analysis is not recommended when the input duty cycle is >50%, as the quality factor will never be zero, and this makes interpretation of the quality factor difficult. However, if the width of the shortest state is the limiting factor for the application, it may be useful to calculate the quality factor using the width of the low-amplitude state, which is less than half a period. This would essentially describe the signal as one with a duty cycle <50%.

2 *Experimental Setup*

The experimental setup is shown in Figure 8, with key components identified. The modulated voltage signal from an 80 MHz function generator was coupled to the external modulation input of the CLD1010LP, which provides current and temperature control for the mounted fiber-pigtailed laser diode. The RF input of the CLD1010LP was not used, as this input was designed for high-speed sinusoidal input signals only. The modulated current output by the controller was used to drive a 980 nm, fiber-pigtailed laser diode, whose optical output was sent through an attenuator and then detected by a 1 GHz DET02AFC FC/PC-coupled photodetector. The signal from the photodetector was input to a 1 GHz oscilloscope.

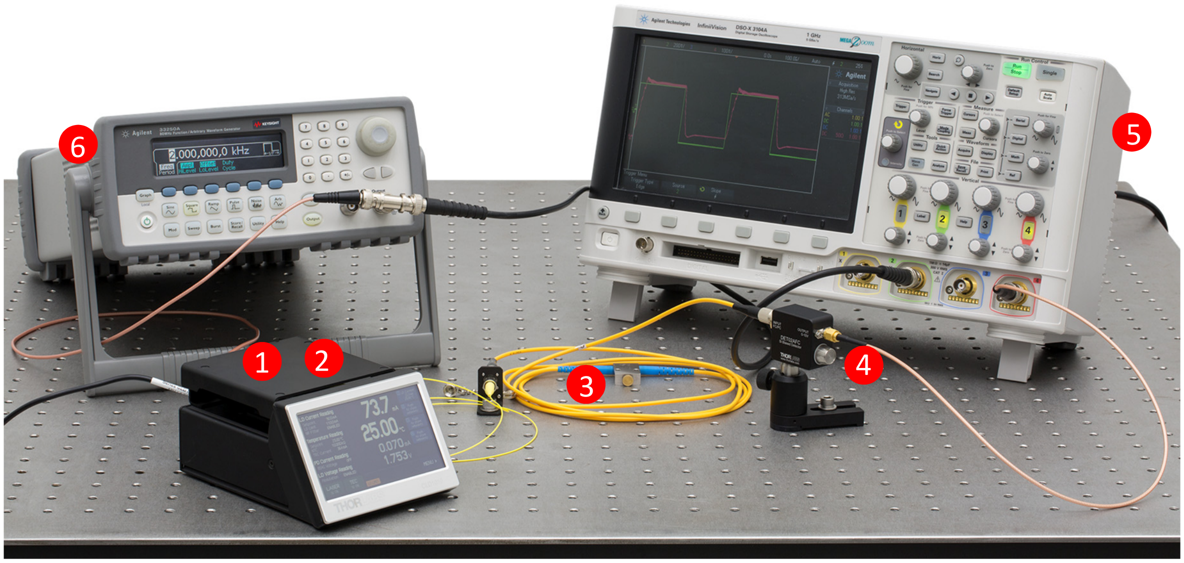


Figure 8 *Experimental Setup used to Modulate a Laser Diode and Measure the Optical Output Signal*

1. CLD1010LP Combined Mount and Current and Temperature Controller for Fiber-Pigtailed Laser Diodes
2. Former Generation LP980-SA80 Laser Diode (Installed in the CLD1010LP and not Visible)
3. VOA980-FC Variable Optical Attenuator
4. DET02AFC 1 GHz FC/PC-Coupled Photodetector
5. 1 GHz Oscilloscope
6. 80 MHz Function Generator

The CLD1010LP was operated in constant current mode and supplied a total current of 140 mA ± 50 mA to the laser diode. The controller also maintained the laser diode at 25 °C. The total driving current was chosen so that it would always exceed the laser diode's 20 mA threshold. The 140 mA constant current component was user-specified and provided by the controller. The modulated current component of approximately ±50 mA resulted from circuitry internal to the controller converting the modulated ±350 mV voltage signal from the function generator to a modulated current signal, with a specified modulation coefficient of 150 mA / V.

The ±350 mV voltage signal amplitude represented only ±5% of the controller's maximum modulation voltage range of ±7 V in constant current mode and was chosen so that the controller would operate under small-signal conditions.

The optical signal from the laser diode was attenuated by a VOA980-FC Single-Mode Variable Optical Attenuator and then coupled to the DET02AFC photodetector. A 1 GHz oscilloscope measured the voltage signals from both the function generator and the DET02AFC. The input impedances of the oscilloscope ports were set to 1 MΩ for the function generator monitor and 50 Ω for the detector measurement.

3 *Results*

Modulated voltage signals from the function generator were input to a system that included the combined laser diode controller and mount, the laser diode, the variable attenuator, and the photodetector. Repetition rates and duty cycles of the input rectangular pulse trains were varied, and the output pulses were detected.

The frequency content of the signals output by both the function generator and the system were analyzed and compared. Amplitudes of the Fourier series coefficients were provided by the oscilloscope, which computed fast Fourier transforms (FFTs) of the measured signals. This was convenient, but the number of calculated coefficients was limited by the oscilloscope's resolution and scan parameters.

Quality factors, discussed in Section 1.3, were also computed for the output pulses. The value of the quality factor was computed using the ideal pulse width and the measured rise time of the output pulse. Measurements of pulse parameters were made manually from the oscilloscope traces.

3.1 Frequency Response of the Laser Diode Driver

The magnitude of the frequency response was measured as described in Section 1.2, and the result is plotted in Figure 9. The function generator provided the set of input sine waves, with frequencies spanning the range from 10 Hz to 1000 kHz. For each input, the peak-to-peak amplitudes of the output voltage waveforms were measured. They were then normalized by the peak-to-peak output signal amplitude measured when the 10 Hz sine wave was input.

From these data, the 3 dB bandwidth of the system was determined to be 750 kHz.

The CLD1010LP combined mount and current and temperature controller had a specified small-signal bandwidth of 300 kHz and was expected to limit the system's bandwidth. However, the measured 3 dB system bandwidth was over twice this value. The difference between the measured and specified 3 dB bandwidths may due to a dependence of the bandwidth on the peak-to-peak, small-signal amplitude. It is possible the amplitude of the input signals used in this work was smaller than that used when developing specifications for the CLD1010LP.

Measuring the 3 dB bandwidth, as was done for this system, may be of interest to a user even when this parameter is provided. The specified 3 dB bandwidth can be a conservatively low value, and the specific operating conditions of an application may affect the absolute value of the 3 dB bandwidth.

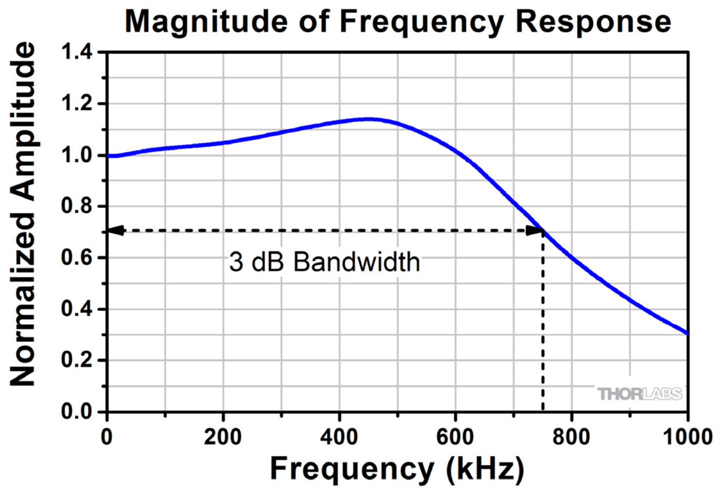


Figure 9 *Frequency response magnitude of the system, found by measuring the peak-to-peak voltages of signals output by the system in response to input sinusoids provided by the function generator. Amplitude values were normalized to the value measured at 10 Hz. The 3 dB system bandwidth extends from DC to the lowest frequency for which the amplitude is 70.7% that of the reference value at 10 Hz.*

3.2 Repetition Rate vs. 3 dB System Bandwidth

The dependence of the output signal's distortion on the relationship between the input signal's repetition rate and the system's 3 dB bandwidth was investigated. To isolate the effects of varying the repetition rate, the duty cycle of the input signal was held constant at 50%, which resulted in a square wave input signal.

Repetition rates of the input square waves were chosen with respect to the 3 dB cutoff frequency, 750 kHz, to limit the number of input signal frequency components within the 3 dB system bandwidth. This intentionally caused distortion of the output signal. Power in each input signal component with a frequency within the 3 dB bandwidth was attenuated by no more than 3 dB, with respect to the reference value measured at 10 Hz. Power in components with frequencies above the 3 dB cutoff frequency was strongly attenuated.

The number of Fourier series expansion terms below the 3 dB cutoff frequency for a particular repetition rate was found using Eq. 3 in Section 1.1. The fundamental frequency of the Fourier series expansion is equal to the repetition rate. The frequency (nf_o) of each sinusoidal expansion term is a harmonic of the fundamental frequency.

The 9X bandwidth rule provides a general guideline for obtaining an output signal that is a reasonable representation of the input square wave. According to the rule, the 3 dB cutoff frequency of the system should be at least a factor of nine higher than the repetition rate of the input square wave. The pulse plotted in Figure 4 (e) includes terms with frequencies up to nine times the repetition rate.

The minimum repetition rate for this work was chosen, according to the 9X rule, to be exactly one ninth of the 3 dB system bandwidth. In this case, the frequencies of the expansion terms with indices up to nine were within the 3 dB system bandwidth. The frequency of the expansion term with index equal to nine was equal to the 3 dB cutoff frequency.

Each row in Table 1 lists the frequencies of an input signal's non-zero Fourier series expansion terms. Frequencies in green cells are within the system's 3 dB bandwidth, and frequencies in white cells are outside this range. The repetition rates of each signal were chosen so that a non-zero term in its Fourier series expansion had a frequency equal to the 3 dB cutoff frequency.

Table 1 *The repetition rates of the five input square waves and the frequencies of the non-zero harmonic terms of the Fourier series expansion with n ≤ 9 are listed. Those frequencies within the 750 kHz system bandwidth are in green.*

	Fundamental (Rep. Rate) n = 1	3rd Harmonic n = 3	5th Harmonic n = 5	7th Harmonic n = 7	9th Harmonic n = 9
Input Signal 1	83.3 kHz	250 kHz	417 kHz	583 kHz	750 kHz
Input Signal 2	107 kHz	321 kHz	535 kHz	750 kHz	964 kHz
Input Signal 3	150 kHz	450 kHz	750 kHz	1050 kHz	1350 kHz
Input Signal 4	250 kHz	750 kHz	1250 kHz	1750 kHz	2250 kHz
Input Signal 5	750 kHz	2250 kHz	3750 kHz	5250 kHz	6750 kHz

Figure 10 shows one period of the waveform measured at the system's output for every input signal. Since every additional term within the system's 3 dB bandwidth improves the output signal's resemblance to a square wave, the output signal most closely resembling a square wave resulted from the input signal with the lowest repetition rate. However, Figure 10 also shows that even when the 3 dB cutoff frequency is only three times the repetition rate, the output signal is distinct from a sine wave. The overshoot on the rising edges and undershoot on the falling edges of the pulses were artifacts, likely from an impedance mismatch between components in the system.

Figure 11 compares the frequency content of the input and output signals. Each grouping in the bar graph shows the amplitudes of a specific Fourier series expansion term for all six signals. Amplitudes for terms with indices through the ninth are included, and the amplitudes of all are normalized with respect to the corresponding term with index one. The amplitudes of the terms are expected to decrease with increasing index due to Eq. 3, which includes a factor of $1/n$ multiplying each term. The bars labeled "Function Generator" represent the coefficients of the input signal and can be used as a reference amplitude for each index.

This bar graph shows that even frequency components below, but approaching, the 3 dB cutoff frequency were attenuated by the system. In addition, some components with frequencies above the cutoff frequency may have non-negligible amplitudes. Examples of this for the 250 kHz repetition rate signal (red) are the amplitudes of its 3rd and 5th index terms, respectively. However, the majority of frequency components above the 3 dB cutoff frequency were strongly attenuated.

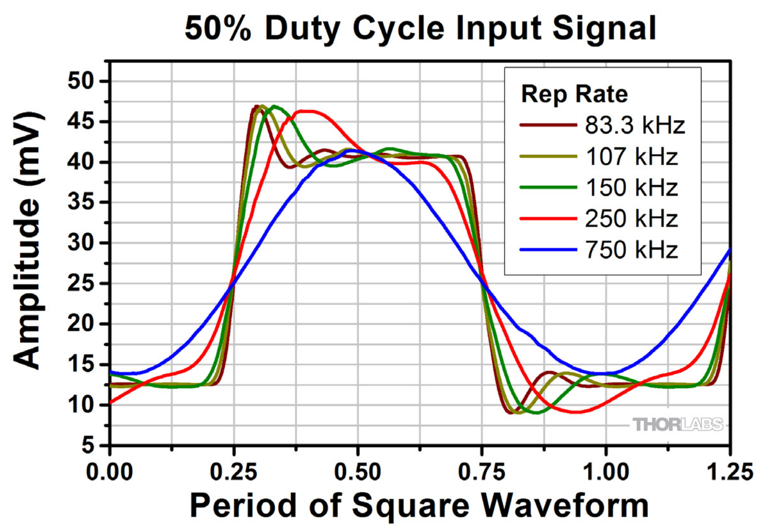


Figure 10 *Measured output waveforms with lower repetition rates have flatter high and low states, as well as steeper rising edges. The overshoots and undershoots when transitioning to high and low states, respectively, were identified as artifacts and affect all but the 750 kHz case.*

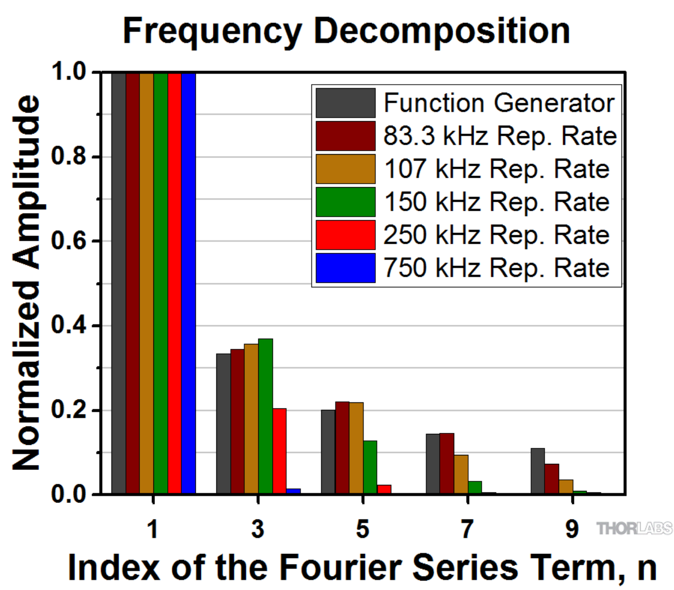


Figure 11 *The frequency content of the waveforms measured directly from the function generator and at the output of the system are compared. Amplitudes were normalized with respect to the amplitude of the Fourier expansion term with index one. The strong attenuation of terms with frequencies ≥ 750 kHz correlates with the distortion of the output waveforms.*

Incrementing the repetition rate by progressing through the five input signals had the effect of pushing one, then another frequency component to the system's 3 dB cutoff frequency. The fewer the number of frequency terms within the 3 dB system bandwidth, the greater the negative impact on the output signal's distortion when the repetition rate was then incremented. For example, compare the 150 kHz repetition rate output waveform, for which three components were within the 3 dB bandwidth, and the 250 kHz repetition rate output waveform, with two. The profile of the 150 kHz repetition rate waveform was similar to that with the 83.3 kHz repetition rate waveform. But, the waveform with the 250 kHz repetition rate had a profile significantly closer in appearance to that of a sinusoidal waveform.

The guideline that the 3 dB system bandwidth should be at least nine times greater than the repetition rate is generally useful, but it may not meet the requirements of a specific application. If a different ratio is needed, it may be helpful to use Fourier series analysis to determine the number of non-zero, higher-index terms of the input waveform that should be preserved in the output signal.

Computing the quality factor offers another approach for predicting signal quality for different repetition rates, given a 3 dB system bandwidth. This approach is described in Section 1.3, and computed quality factors for these waveforms are presented and discussed in Section 3.4.

3.3 Modulated Output vs. Input Signal Duty Cycle

The distortion of the output pulses depended on the duty cycle of the input signal, as well as on its repetition rate. Input signals with 50% duty cycles were considered exclusively in Section 3.2. In this section, output signal distortion was investigated for input rectangular pulse trains with different duty cycles. The dependence of the output signal's duty cycle on the system's 3 dB cutoff frequency, the repetition rate of the input signal, and the duty cycle of the input signal was also investigated.

Input signals consisting of periodic rectangular pulse trains first with 20%, and then with 80%, duty cycles were input to the system. Selected output signals are shown in Figure 12 and Figure 13. For comparison, Figure 10 gives examples of output waveforms resulting from input signals with 50% duty cycles.

These plots show that repetition rates providing acceptable output signal quality when the input signal's duty cycle was 50% may not provide acceptable signal quality if the input signal's duty cycle is changed. For example, when the input signal had an 83.3 kHz repetition rate, the profiles of the output waveforms' high states varied from rounded to approximately flat as the duty cycle of the input signal was varied from 20% to 80%.

This was due to the duty cycle affecting the ratio of rise time to high-state duration. Reducing the duty cycle while holding the repetition rate constant resulted in a shorter-duration high state. A consequence was that the output signal in the 20% duty cycle case had less time to stabilize at the high state before transitioning to the low state. This resulted in output pulses for the 20% duty cycle case that were significantly more rounded, for each repetition rate, than those measured for the other duty cycle cases.

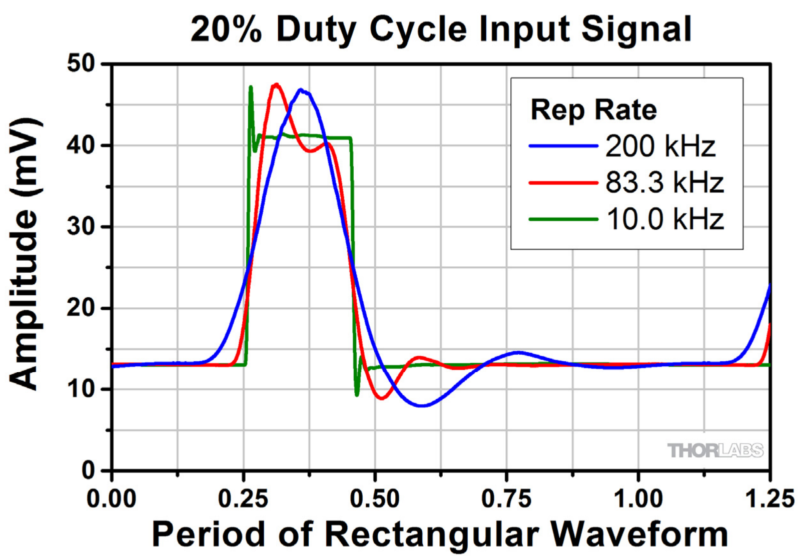


Figure 12 *Waveforms output for input rectangular pulse trains with 20% duty cycles and repetition rates up to 200 kHz are plotted. When just the high state is considered, only the 10.0 kHz repetition rate output signal had a profile easily recognizable as rectangular.*

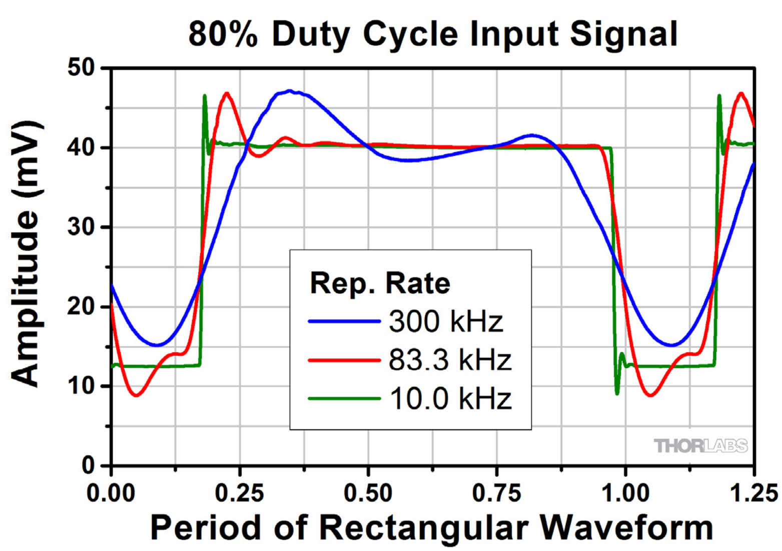


Figure 13 *Waveforms output for input rectangular pulse trains with 80% duty cycles and repetition rates up to 300 kHz, essentially an inverse of the 20% duty cycle case, are plotted. When just the high state is considered, both the 83.3 kHz and 10.0 kHz repetition rate output signals had recognizably rectangular profiles.*

Acceptable combinations of the input signal's repetition rate, the input signal's duty cycle, and the system's 3 dB bandwidth depend on the application. If the application requires only the detection of a pulse, rather than the preservation of the rectangular profile of the pulse, a higher repetition rate may be acceptable for a wide range of duty cycles. If the application specifies minimum acceptable output signal quality over both the high and low state durations, the repetition rate may need to be reduced to meet the requirements for the shorter-duration state.

Figure 14 plots the duty cycles of the signals output by the 750 kHz bandwidth system as functions of the input signal's duty cycle and repetition rate. The dependence of the output signal's duty cycle on the system's 3 dB bandwidth can be explained by considering the Fourier series expansion of a periodic rectangular pulse train. The shorter-duration states were narrowed and sharpened when more higher-frequency terms were included in the waveform. Increasing the repetition rate caused more higher-index terms to be strongly attenuated, as their frequencies exceed the system's 3 dB cutoff frequency. After enough higher order terms were attenuated, the fundamental sinusoidal term began to dominate. As sinusoids have a 50% duty cycle, the duty cycle of the output waveform converged to 50% as the repetition rate approached the system's 3 dB cutoff frequency. For reference, see Figure 2.

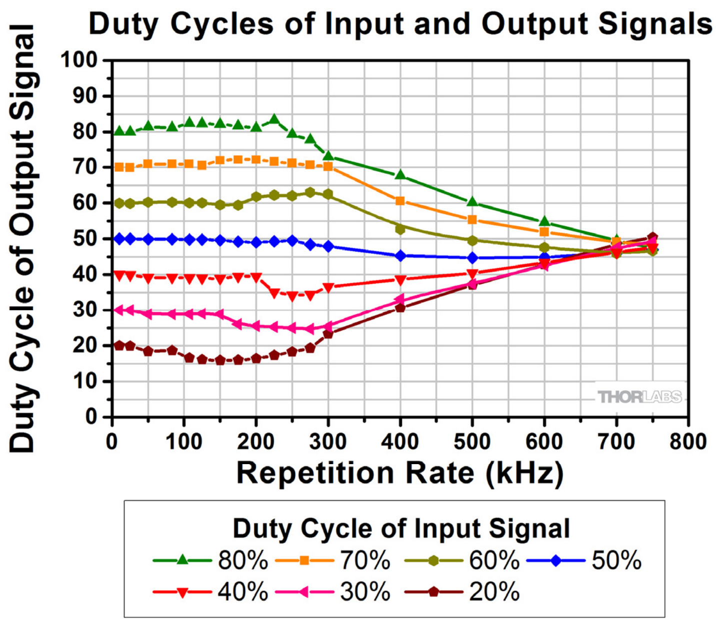


Figure 14 *The measured duty cycles of the output signals approached 50% as the repetition rates increased due to the attenuation of the higher frequency terms, which are required to narrow and sharpen the shorter-duration state.*

3.4 Quality Factors

Quality factors, introduced in Section 1.3, are used to numerically rate whether an output pulse shape is closer to being rectangular or sinusoidal. While Fourier series analysis can be applied to any periodic signal, the quality factor approach is targeted to periodic rectangular pulse trains. Quality factors are functions of the repetition rate and duty cycles of the input signal, as well as the bandwidth of the system.

An ideal rectangular pulse train is assumed to be the desired output waveform. Pulses that instantaneously transition between low and high state amplitudes, a key characteristic of ideal rectangular pulses, and have the width of an ideal rectangular pulse, have quality factors of one. Pulses that require the full duration of an ideal rectangular pulse to transition between low and high state amplitudes have quality factors of zero. If the input signal has a 50% duty cycle, a zero quality factor indicates the output waveform is a pure sinusoid. If the duty cycle is <50% and the quality factor is zero, the corresponding output waveform will not be sinusoidal. It will instead exhibit periodically repeating features identifiable as pulses.

Quality factors were calculated for output pulses resulting from input signals with 50%, 20%, and 80% duty cycles. The quality factors were plotted with respect to repetition rate in Figure 15. The solid red circles are values corresponding to the waveforms plotted in Figure 10, Figure 12, and Figure 13.

The blue curve corresponds to the 50% duty cycle case. As discussed in Section 3.2, the minimum repetition rate of the square wave signal input, 83.3 kHz, was chosen so that the system's 3 dB cutoff frequency, 750 kHz, would be nine times higher. This choice was guided by the 9X bandwidth rule, which states that the output signal will be a reasonable representation of the input square wave when the system's 3 dB cutoff frequency is at least nine times higher than the repetition rate. The 0.87 quality factor calculated for the 83.3 kHz repetition rate is close to the maximum quality factor of one, indicating that the pulse shapes of this output signal and the ideal square wave compare well. This result supports the use of the 9X rule.

It was also noted in Section 3.2 that a threefold difference, corresponding to a 250 kHz repetition rate, resulted in an output waveform identifiable as a square wave. This could have been predicted by referencing its 0.58 quality factor from Figure 15.

Quality factors for the 20% duty cycle case were consistent with the observation, made in Section 3.3, that reducing the duty cycle while maintaining the repetition rate increases pulse distortion. The 0.68 quality factor value for the 83.3 kHz repetition rate corresponds to an identifiably square, but more distorted, pulse shape than in the 50% duty cycle, 83.3 kHz repetition rate case. Referencing Figure 15, a repetition rate around 33 kHz would have been required to obtain an output pulse with the 0.87 quality factor of the 50% duty cycle, 83.3 kHz repetition rate case. The quality factor was zero for a repetition rate near 250 kHz. Quality factors corresponding to higher repetition rates were negative and are not include on this plot.

When the duty cycle of the input signal is >50%, it is challenging to interpret the quality factors. As the minimum quality factor varies with duty cycle and is positive, it is difficult to compare these values with those found for other duty cycle cases. In addition, the quality factor rates the shape of the pulse, but does not assess the worse distortion of the low-state segments. To address these concerns, the quality factor can be computed using the width of the low-state, rather than high-state, of the ideal rectangular pulse train. The data for the 80% case were calculated both ways. The data for the 80% High curve were calculated using the width of the high state of the rectangular pulse, and the data for the 80% Low curve were calculated using the width of the low state. The 80% Low data resembled the 20% duty cycle data, as expected.

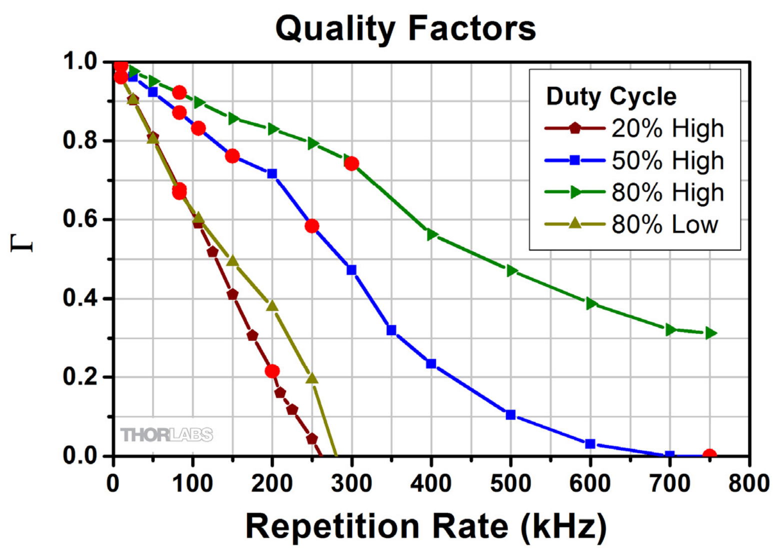


Figure 15 *Periodic rectangular pulse trains were input to a system with a 3 dB cutoff of 750 kHz, and the quality factors of the output waveforms were computed. The two curves for the 80% case correspond to quality factors computed using the ideal high state and low state widths. The red circles correspond to data points for the waveforms plotted in Figure 10, Figure 12, and Figure 13. The lines connecting the data points are guides for the eye.*

The data plotted in Figure 15 show the influence of the system's 3 dB bandwidth as well as the input signal's pulse width, repetition rate, and duty cycle on the quality factor. The relationship among these parameters follows from the inverse relationship between the width of the pulses and the bandwidth required to transmit them with reasonable fidelity.

Pulse widths decrease as the repetition rate increases, resulting in the negative slopes of the quality factor curves. Pulse widths also decrease as the duty cycle decreases, so that decreasing the duty cycle results in curves whose negative slopes are steeper.

Faster transition times between low and high states are required to limit the output pulse distortion when pulse widths are shorter. One way to adjust the pulse width is to change the repetition rate. Another way is to maintain the repetition rate, but to change the duty cycle.

4 *Summary*

The effect of repetition rate, duty cycle, and 3 dB system bandwidth on the distortion of the output signal was investigated. Waveforms input to a laser diode controller were periodic, rectangular pulse trains from a function generator with duty cycles between 20% and 80% and repetition rates from 10 Hz to 750 kHz. The modulated optical output signals from the laser diode were detected using a photodiode. An oscilloscope was used to record the output signals, which were characterized by calculating the Fourier series coefficients and computing quality factors from the ideal pulse widths and measured rise times of the output pulses.

Fourier series analysis, which can be applied to any periodic signal, was used to model the output waveforms and analyze the frequency content of measured waveforms. Analysis showed that increasing the repetition rate, which is equal to the first harmonic frequency of the Fourier series expansion, increases the frequencies of all terms in the expansion.

This effect was shown to affect the relationship between the duty cycles of the input and output signals, when the duty cycle of the input signal is not 50%. In this case, increasing the repetition rate of the input signal results in an output signal whose duty cycle is closer to 50% than expected. The cause of this was shown to be the greater attenuation of a larger number of higher-index terms with each increase in repetition rate, since this increased the frequencies of all input signal components relative to the system's 3 dB cutoff frequency.

A quality factor, targeted to nominally rectangular pulse shapes, was derived to be used as a pulse shape assessment or prediction tool. It numerically rates pulse shape as being more similar to either an ideal rectangular or sinusoidal profile. The quality factor scale has a maximum value of one, corresponding to an ideal rectangular profile, and a minimum value that depends on the duty cycle of the input waveform. Quality factor values vary with 3 dB system bandwidth, input signal repetition rate, and input signal duty cycle. For input signals with duty cycles ≤50%, a quality factor greater than approximately 0.5 indicates the output pulse shape is more similar to a rectangular profile.

Both Fourier analysis and quality factor approaches were used to investigate and confirm the 9X bandwidth rule for square wave transmission, which recommends the 3 dB system bandwidth be at least a factor of nine higher than the repetition rate of the input signal. Fourier analysis showed that this ninefold difference would result in the frequencies of the five lowest-index, non-zero harmonic terms being within the 3 dB system bandwidth. When this rule was followed, the output signal was observed to be a reasonable representation of the input square wave. The 0.87 quality factor found for this case also compared well with the ideal value of one, on a scale of zero to one. The 9X rule was also found to be a useful guideline for 20% duty cycle pulses; the quality factor for the 83.3 kHz repetition rate pulse was 0.68.

A threefold difference between the repetition rate and the 3 dB system bandwidth preserves the three lowest-index terms of the Fourier series expansion in the output signal. The terms with indices one and three had non-zero amplitudes when the input signal was a square wave, and the output waveform had an identifiably square pulse shape. The 0.58 quality factor for this case supports the use of a 0.5 quality factor value to define the threshold above which pulse profiles are more similar to rectangular. When the duty cycle of the input rectangular pulse train was 20%, a threefold difference was not sufficient to provide output pulses with recognizably rectangular profiles. Achieving a recognizably rectangular profile would require including additional higher-frequency terms in the signal, which would improve the definition of its narrow features.